Mechanics of Advanced Composite Structures 2 (2015) 45-53

Semnan University

Mechanics of Advanced Composite Structures

journal homepage: http://macs.journals.semnan.ac.ir

Static Flexure of Soft Core Sandwich Beams using Trigonometric Shear Deformation Theory

A.S. Sayyad[a*], Y.M. Ghugal[b]

[a] *Department of Civil Engineering, SRES's College of Engineering, Savitribai Phule Pune University, Kopargaon, Maharashtra, India*
[b] *Department of Applied Mechanics, Government College of Engineering, Karad, Maharashtra, India*

PAPER INFO

Paper history:
Received 17 April 2015
Received in revised form 6 November 2015
Accepted 7 November 2015

Keywords:
Laminated beam
Soft core
Sandwich beam
Flexure
Trigonometric shear deformation theory

ABSTRACT

This study deals with the applications of a trigonometric shear deformation theory considering the effect of the transverse shear deformation on the static flexural analysis of the soft core sandwich beams. The theory gives realistic variation of the transverse shear stress through the thickness, and satisfies the transverse shear stress free conditions at the top and bottom surfaces of the beam. The theory does not require a problem-dependent shear correction factor. The governing differential equations and the associated boundary conditions of the present theory are obtained using the principle of the virtual work. The closed-form solutions for the beams with simply supported boundary conditions are obtained using Navier solution technique. Several types of sandwich beams are considered for the detailed numerical study. The axial displacement, transverse displacement, normal and transverse shear stresses are presented in a non-dimensional form and are compared with the previously published results. The transverse shear stress continuity is maintained at the layer interface, using the equilibrium equations of elasticity theory.

1. Introduction

Sandwich beam is a special form of laminated composite beam which has stiff face sheets and light weight but thick core. The modulus of the core material is significantly lower than that of the face sheets. The main benefit of using the sandwich concept in the structural components is its high bending stiffness and high strength to weight ratio. In addition, the sandwich constructions are much preferred to conventional materials because of their superior mechanical and durability properties. Due to these properties the composite sandwich structures have been widely used in the automotive, aerospace, marine and other industrial applications. Therefore, the analytical study of the sandwich beams becomes increasingly important.

Since the Classical Beam Theory (CBT) neglects the effect of the shear deformation and the First-order Shear Deformation Theory (FSDT) of Timoshenko [1] requires a shear correction factor, these theories are not suitable for the analysis of the laminated composite and the sandwich beams. These limitations of CBT and FSDT have led to the development of the Higher-order Shear Deformation theories (HSDTs) taking into account the effect of the transverse shear deformation, obviating the need of a shear correction factor.

The beam theories can be developed by expanding the displacements in power series of the coordinate normal to the neutral axis. In principle, the theories developed by this means can be made as accurate as desired simply by including the sufficient number of terms in the series. These higher-order theories are cumbersome and computationally more

* Corresponding author, Tel.: +91-9763567881

E-mail address: attu_sayyad@yahoo.co.in

demanding, because with an additional power of the thickness coordinate, an additional dependent variable is introduced into the theory. It has been noted by Lo et al. [2, 3] that due to the higher-order terms included in their theory, it has become inconvenient to use. This observation is more or less true for many other higher-order theories as well. Thus, there is a wide scope to develop a simple to use higher-order beam or plate theory.

Several theories have been proposed by researchers in the last two decades. Among many theories, some of the well-known theories are the parabolic shear deformation theories [4-5], the trigonometric shear deformation theory [6], the hyperbolic shear deformation theory [7] and the exponential shear deformation theory [8]. Recently, these theories are accounted into a unified shear deformation theory developed by Sayyad [9] and Sayyad et al. [10]. In accordance with Reddy's third-order shear deformation theory, Sayyad [11] has developed the refined theories and applied them for the static and vibration analysis of the isotropic beams.

Mechab et al. [12] studied the deformations of the short composite beams using the refined theories. Carrera and Giunta [13] developed refined beam theories based on a unified formulation. Carrera et al. [14, 15] carried out the static and free vibration analysis of laminated beams using polynomial, trigonometric, exponential and zig-zag theories. Giunta et al. [16] presented a thermo-mechanical analysis of isotropic and composite beams via collocating with radial basis functions. Chakrabarti et al. [17] and Chalak et al. [18] carried out a finite element analysis for the bending, buckling and free vibration of the soft core sandwich beams. Gherlone et al. [19] developed C^0 beam elements based on the refined zig-zag theory for the multilayered laminated composite and sandwich beams.

In the class of Trigonometric Shear Deformation Theories (TSDTs), the shear deformation is assumed to be trigonometric with respect to the thickness coordinate. These theories are accounted cosine distribution of transverse shear stress. The TSDTs are taking into account the kinematics of higher-order theories more effectively without loss of the physics of the problem. Some of the well-known articles on trigonometric theories are published by Touratier [6], Shimpi and Ghugal [20], Ghugal and Shinde [21], Arya et al. [22], Sayyad and Ghugal [23], Mantari et al. [24], Ferreira et al. [25], Zenkour [26] and Sayyad et al. [27]. Recently, Dahake and Ghugal [28, 29] and Ghugal and Dahake [30] have applied the trigonometric shear deformation theory for the bending analysis of the single-layer isotropic beams with various boundary conditions using general solution technique.

In the current study, a trigonometric shear deformation theory is applied for the bending analysis of the laminated composite and the soft core sandwich beams. The theory involves three unknowns. The theory satisfies the transverse shear stress free conditions at the top and bottom surfaces of the beam and does not require shear correction factor. The governing equations are obtained using the principle of the virtual work. The closed-form solutions for the beam with simply supported boundary conditions are obtained using Navier solution technique. The displacements and stresses of three different types of lamination scheme are obtained.

The exact elasticity solution for the three-layered ($0^0/90^0/0^0$) laminated composite developed by Pagano [31] is used as a basis for the comparison of the present results. However, the exact elasticity solutions for the three-layered (0^0/core/0^0) and five-layered ($0^0/90^0$/core/$90^0/0^0$) sandwich beams are not available in the literature. Authors have generated the numerical results using FSDT of Timoshenko [1], HSDT of Reddy [5] and CBT being not available. It is found that the present results are in excellent agreement with those of HSDT, FSDT, CBT and exact elasticity solution.

2. Sandwich Beam under Consideration

Consider a beam of length 'L' along x direction, width 'b' along y direction and thickness 'h' along z direction. The coordinate system and geometry of the beam under consideration are shown in Fig. 1. The beam consists of the face sheets at the top and bottom surfaces and the middle portion is made up of a soft core.

The beam is bounded in the region $0 \le x \le L$, $-b/2 \le y \le b/2$, $-h/2 \le z \le h/2$ in Cartesian coordinate system. u and w are the displacements in x and z directions, respectively.

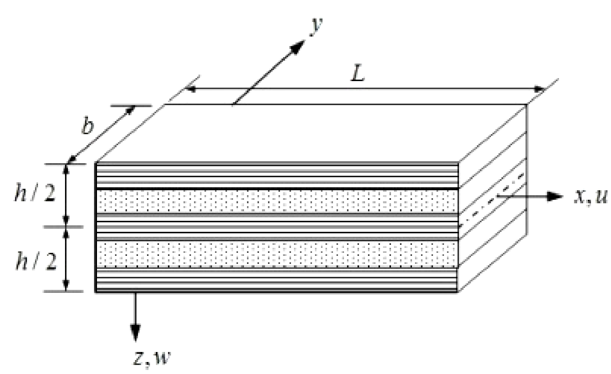

Figure 1. The beam geometry and the coordinate system

2.1. *The Assumptions made in the Theoretical Formulation*

In the present equivalent single-layer trigonometric shear deformation theory, the theoretical formulation is based on the six following assumptions:

1) The axial displacement u in x direction consists of two parts including (a) a displacement component analogous to the displacement in the classical beam theory and (b) a displacement component due to the shear deformation which is assumed to be sinusoidal in nature with respect to the thickness coordinate.
2) The transverse displacement w in the z direction is assumed to be a function of the x coordinate only.
3) The beam is made up of 'N' number of layers which are perfectly bonded together.
4) One dimensional Hooke's law is used.
5) The beam is subjected to the lateral load only.
6) The body forces are ignored.

2.2. *The Kinematics of the Present Theory*

Based on the above mentioned assumptions, the displacement field of the present trigonometric shear deformation theory is written as:

$$u(x,z)=u_0(x)-z\,w_{0,x}+(h/\pi)(\sin\pi\bar{z})\phi(x)$$
$$w(x)=w_0(x) \tag{1}$$

where u and w are the displacements in x and z directions, respectively and $\bar{z}=z/h$ is the thickness coordinate. u_0, w_0 and ϕ are the unknown functions to be determined. The normal and shear strains obtained within the framework of the linear theory of elasticity are as follows:

$$\varepsilon_x=u_{,x}=u_{0,x}-zw_{0,xx}+f(\bar{z})\phi_{,x}$$
$$\gamma_{zx}=u_{,z}+w_{,x}=g(\bar{z})\phi \tag{2}$$

where

$$f(\bar{z})=(h/\pi)\sin(\pi\bar{z}) \text{ and } g(\bar{z})=\cos(\pi\bar{z}) \tag{3}$$

'$,_x$' represents the derivative with respect to x.

2.3. *The constitutive relations*

The normal and transverse shear stresses are obtained using one-dimensional constitutive relations. These relations for the k^{th} layer of the beam are given by the following equations:

$$\sigma_x^k=Q_{11}^k\,\varepsilon_x=Q_{11}^k\left[u_{0,x}-zw_{0,xx}+(h/\pi)(\sin\pi\bar{z})\phi_{,x}\right]$$
$$\tau_{zx}^k=Q_{55}^k\,\gamma_{zx}=Q_{55}^k\,g(\bar{z})\phi \tag{4}$$

where Q_{11}^k and Q_{55}^k are the stiffness coefficients of the k^{th} layer of the beam and are defined as follows:

$$Q_{11}^k=E_{11}^k \text{ and } Q_{55}^k=G_{13}^k$$

where E_{11}^k is the Young's modulus and G_{13}^k is the shear modulus of k-th layer of the beam.

3. Governing Equations and Boundary Conditions

In order to derive the governing equations, the principle of the virtual work is used.

$$b\int_0^L\int_{-h/2}^{h/2}\left(\sigma_x\delta\varepsilon_x+\tau_{zx}\delta\gamma_{zx}\right)dz\,dx-\int_0^L q(x)\,\delta w\,dx=0 \tag{5}$$

Using the expressions for strains from Eq. (2) and stresses from Eq. (4), the Eq. (5) can be written as:

$$\begin{aligned}&b\int_{-h/2}^{h/2}\int_0^L Q_{11}^{(k)}\big[u_{0,x}\,\delta u_{0,x}-z\,u_{0,x}\delta w_{0,xx}+f(\bar{z})u_{0,x}\delta\phi_{,x}\\&-z\,w_{0,xx}\,\delta u_{0,x}+z^2\,w_{0,xx}\,\delta w_{0,xx}-z\,f(\bar{z})\,w_{0,xx}\delta\phi_{,x}+\\&f(\bar{z})\phi_{,x}\,\delta u_{0,x}-z\,f(\bar{z})\phi_{,x}\delta w_{0,xx}+f(\bar{z})^2\phi_{,x}\delta\phi_{,x}\big]dx\,dz\\&+b\int_{-h/2}^{h/2}\int_0^L Q_{55}^{(k)}\,g(\bar{z})^2\,\phi\,\delta\phi\,dx\,dz-\int_0^L q\,\delta w_0\,dx=0\end{aligned} \tag{6}$$

Integrating Eq. (6) with respect to the z-direction, Eq. (6) can be simplified as:

$$\begin{aligned}&\int_0^L\big[A_{11}\,u_{0,x}\,\delta u_{0,x}-B_{11}\,u_{0,x}\delta w_{0,xx}+C_{11}\,u_{0,x}\delta\phi_{,x}\\&-B_{11}\,w_{0,xx}\,\delta u_{0,x}+D_{11}\,w_{0,xx}\,\delta w_{0,xx}-E_{11}\,w_{0,xx}\,\delta\phi_{,x}\\&+C_{11}\,\phi_{,x}\,\delta u_{0,x}-E_{11}\,\phi_{,x}\delta w_{0,xx}+F_{11}\,\phi_{,x}\delta\phi_{,x}\\&+G_{55}\,\phi\,\delta\phi-q\,\delta w_0\big]dx=0\end{aligned} \tag{8}$$

where A_{ij}, B_{ij}, etc. are the beam stiffnesses as defined below:

$$\begin{aligned}&(A_{11},B_{11},C_{11})=b\sum_{k=1}^{N}Q_{11}^{(k)}\int_{z_k}^{z_{k+1}}\left(1,z,f(\bar{z})\right)dz,\\&(D_{11},E_{11},F_{11})=b\sum_{k=1}^{N}Q_{11}^{(k)}\int_{z_k}^{z_{k+1}}\left(z^2,z\,f(\bar{z}),f(\bar{z})^2\right)dz,\\&G_{55}=b\sum_{k=1}^{N}Q_{55}^{(k)}\int_{z_k}^{z_{k+1}}g(\bar{z})^2\,dz.\end{aligned} \tag{9}$$

Integrating Eq. (8) by the parts and setting the coefficients of δu_0, δw_0 and $\delta\phi$ equal to zero, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing equations of the beam are as follow:

$$-A_{11}\,u_{0,xx}+B_{11}\,w_{0,xxx}-C_{11}\,\phi_{,xx}=0 \tag{10}$$
$$-B_{11}\,u_{0,xxx}+D_{11}\,w_{0,xxxx}-E_{11}\,\phi_{,xxx}=q(x) \tag{11}$$
$$-C_{11}\,u_{0,xx}+E_{11}\,w_{0,xxx}-F_{11}\,\phi_{,xx}+G_{55}\,\phi=0 \tag{12}$$

The associated consistent natural boundary conditions at the ends $x = 0$ and $x = L$ are as follows:

$$N_x=0 \quad\text{or}\quad u_0 \text{ is prescribed} \tag{13}$$
$$\frac{dM_x^c}{dx}=0 \quad\text{or}\quad w_0 \text{ is prescribed} \tag{14}$$
$$M_x^c=0 \quad\text{or}\quad w_{0,x} \text{ is prescribed} \tag{15}$$
$$M_x^s=0 \quad\text{or}\quad \phi \text{ is prescribed} \tag{16}$$

where the resultants N_x, M_x^c, M_x^s are defined using the following equations:

$$N_x = \int_{-h/2}^{+h/2} \sigma_x \, dz = A_{11}u_{0,x} - B_{11}w_{0,xx} + C_{11}\phi_{,x} \quad (17)$$

$$M_x^c = \int_{-h/2}^{+h/2} \sigma_x \, z \, dz = B_{11}u_{0,x} - D_{11}w_{0,xx} + E_{11}\phi_{,x} \quad (18)$$

$$M_x^s = \int_{-h/2}^{+h/2} \sigma_x \, f(\bar{z}) \, dz = C_{11}u_{0,x} - E_{11}w_{0,xx} + F_{11}\phi_{,x} \quad (19)$$

Thus, the variationally consistent governing differential equations and boundary conditions are obtained.

4. A static Flexure of Sandwich Beam

The Navier solution satisfies the governing differential equation and boundary conditions when the beam is simply supported at the ends. Therefore, a static flexural analysis of the simply supported laminated composite and the soft core sandwich beams subjected to transverse distributed load has been carried out using Navier solution technique. According to this technique, the following solution form for unknown functions u_0, w_0 and ϕ is assumed.

$$u_0(x) = \sum_{m=1,3,5}^{\infty} u_m \cos \alpha x, \quad w_0(x) = \sum_{m=1,3,5}^{\infty} w_m \sin \alpha x, \quad \phi(x) = \sum_{m=1,3,5}^{\infty} \phi_m \cos \alpha x \quad (20)$$

where $\alpha = m\pi / L$ and u_m, w_m and ϕ_m are the unknown Fourier coefficients to be determined for each m value. A beam of length L and thickness h is considered. The transverse load acting on the top surface of the beam is expanded in the following form:

$$q(x) = \sum_{m=1} q_0 \sin \alpha x: \quad \text{Sinusoidal Load} \quad (21)$$

$$q(x) = \sum_{m=1,3,5}^{\infty} \frac{4q_0}{m\pi} \sin \alpha x: \quad \text{Uniform Load} \quad (22)$$

where q_o is the maximum intensity of the load. Substituting the solution form from Eq. (20) and transverse load from Eqs. (21) and (22) into the three governing Eqs. (10)-(12), leads to the following set of simultaneous equatios.

$$\begin{aligned} &\left(A_{11}\alpha^2\right)u_m - \left(B_{11}\alpha^3\right)w_m + \left(C_{11}\alpha^2\right)\phi_m = 0 \\ &-\left(B_{11}\alpha^3\right)u_m + \left(D_{11}\alpha^4\right)w_m - \left(E_{11}\alpha^3\right)\phi_m = q_m \\ &\left(C_{11}\alpha^2\right)u_m - \left(E_{11}\alpha^3\right)w_m + \left(F_{11}\alpha^2 + G_{55}\right)\phi_m = 0 \end{aligned} \quad (23)$$

Eq. (23) can be solved to obtain the Fourier coefficients u_m, w_m and ϕ_m. Further, the final expressions for displacements and stresses are obtained using Eqs. (1)-(4).

5. Illustrative Examples and Numerical Results

To prove the efficacy of the present theory, it is applied to the flexural analysis of the following examples on the laminated composite and soft core sandwich beams.

***Example* 1:**

A static flexure of the three-layered ($0^0/90^0/0^0$) laminated composite beams, as shown in Fig. 2 (a).

***Example* 2:**

A static flexure of the three-layered (0^0/core/0^0) soft core sandwich beams, as shown in Fig. 2 (b).

***Example* 3:**

A static flexure of the five-layered ($0^0/90^0$/core/$90^0/0^0$) soft core sandwich beams, as shown in Fig. 2 (c).

The following material properties are used in the above examples:

Example 1:

0^0 layer: Q_{11}= 25×10^6 psi, Q_{55}= 0.5×10^6 psi

90^0 layer: Q_{11}= 1.0×10^6 psi, Q_{55}= 0.2×10^6 psi

Examples 2 and 3:

Face sheets (0^0): Q_{11}= 25×10^6 psi, Q_{55}= 0.5×10^6 psi

Face sheets (90^0): Q_{11}= 1.0×10^6 psi, Q_{55}= 0.2×10^6 psi

Core: Q_{11}= 0.04×10^6 psi, Q_{55}= 0.06×10^6 psi.

The numerical results obtained for the displacements and stresses at the critical points are presented in the following non-dimensional form (Take E_3 = 1).

$$\bar{u}(0,-h/2) = \frac{u\,b\,E_3}{q_0 h}, \; \bar{w}(L/2,0) = \frac{w100\,h^3 E_3}{q_0 a^4}, \quad \bar{\sigma}_x(0,-h/2) = \frac{\sigma_x\, b}{q_0}, \quad \bar{\tau}_{zx}(0,0) = \frac{\tau_{zx}\, b}{q_0} \quad (24)$$

Example 1:

In this example, the bending response of the simply supported three-layered (0°/90°/0°) laminated composite beam is investigated as shown in Fig. 2 (a). The numerical results for the non-dimensional displacements and stresses are presented in Tables 1 and 2. For the comparison purpose, the numerical results are specially generated using the Higher-order Shear Deformation Theory (HSDT) of Reddy [5], the First-order Shear Deformation Theory (FSDT) of Timoshenko [1] and the Classical Beam Theory (CBT). The examination of Tables 1 and 2 reveals that when the laminated composite beam is subjected to the sinusoidal/uniform load, the displacements and normal stresses are in excellent agreement with those of Higher-order Shear Deformation Theory (HSDT) of Reddy [5]. The transverse shear stress is obtained using the equilibrium equation of the elasticity theory with the shear stress continuity at the layer interface.

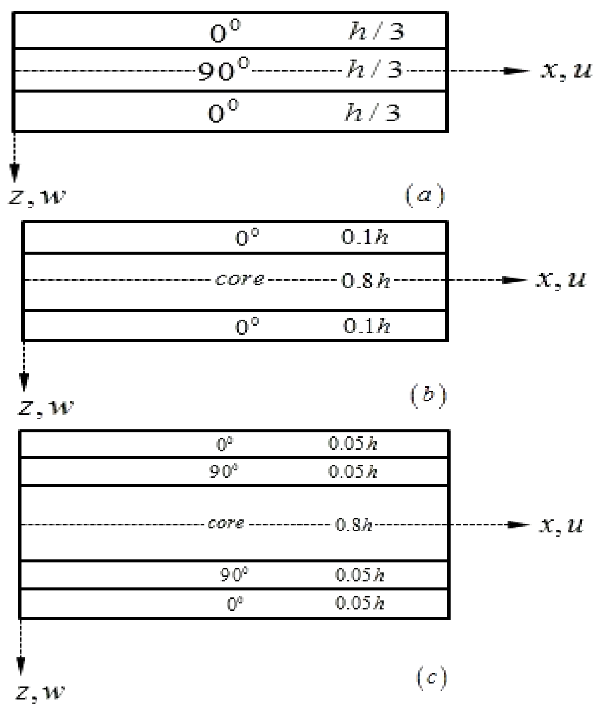

Figure 2. The lamination scheme and the thickness coordinate for the simply supported beams

Table 1: The comparison of the axial displacement ($\bar{u}$), the transverse displacement ($\bar{w}$), the normal stress ($\bar{\sigma}_x$), and the transverse shear stress ($\bar{\tau}_{zx}$), for three-layered (0°/90°/0°) laminated composite beam subjected to the sinusoidal load

L/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}$
100	Present	8037.5	0.5148	6312.6	44.22
	HSDT [5]	8034.9	0.5146	6310.6	44.27
	FSDT [1]	8025.7	0.5135	6303.3	44.22
	CBT	8025.7	0.5109	6303.3	44.22
	Exact [31]	8040.0	0.5153	6315	44.15
10	Present	9.019	0.8836	70.853	4.320
	HSDT [5]	8.939	0.8751	70.212	4.330
	TSDT [27]	9.016	0.8828	70.836	4.322
	FSDT [1]	8.025	0.8149	63.033	4.422
	CBT	8.025	0.5109	63.033	4.420
	Exact [31]	9.105	0.8800	71.300	4.200
4	Present	0.892	2.7340	17.540	1.532
	HSDT [5]	0.865	2.7000	17.006	1.557
	TSDT [27]	0.891	2.7252	17.500	1.528
	FSDT [1]	0.514	2.4107	10.085	1.769
	CBT	0.514	0.5109	10.085	1.769
	Exact [31]	0.915	2.8870	17.880	1.425

The detailed procedure to obtain this stress using equilibrium equation is given by Sayyad et al. [27]. The through thickness distributions of axial displacement, normal stress and transverse shear stress are shown in Figs. 3-5.

Example 2:

This example investigates the bending response of the three-layered (0°/core/0°) soft core sandwich beams as shown in Fig. 2 (b). The beam has thin top and bottom face sheets of thickness 0.1h each and thick core of thickness 0.8h. The comparison of results for the beam subjected to the sinusoidal load is shown in Table 3.

Table 2: The comparison of the axial displacement ($\bar{u}$), the transverse displacement ($\bar{w}$), the normal stress ($\bar{\sigma}_x$), and the transverse shear stress ($\bar{\tau}_{zx}$), for the three-layered (0°/90°/0°) laminated composite beam subjected to the uniform load

L/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}$
100	Present	10386.3	0.6528	7786.5	68.239
	HSDT [5]	10382.8	0.6527	7784.2	68.243
	FSDT [1]	10368.6	0.6518	7776.7	68.387
	CBT	10368.6	0.6480	7776.7	68.387
10	Present	11.844	1.108	85.68	6.042
	HSDT [5]	11.733	1.098	85.03	6.090
	FSDT [1]	10.368	1.023	77.76	6.838
	CBT	10.368	0.648	77.76	6.838
4	Present	1.195	3.413	20.30	2.629
	HSDT [5]	1.161	3.368	19.67	2.795
	FSDT [1]	0.663	2.991	12.44	2.735
	CBT	0.663	0.648	12.44	2.735

Table 3: The comparison of the axial displacement ($\bar{u}$), the transverse displacement ($\bar{w}$), the normal stress ($\bar{\sigma}_x$), and the transverse shear stress ($\bar{\tau}_{zx}$), for the three-layered (0°/core/0°) soft core sandwich beam subjected to the sinusoidal load

L/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}$
4	Present	1.7678	10.091	34.710	1.3732
	HSDT [5]	1.7413	10.047	34.189	1.3681
	FSDT [1]	1.0134	5.2868	19.898	1.4106
	CBT	1.0134	1.0081	19.898	1.4106
10	Present	17.758	2.4887	139.47	3.5094
	HSDT [5]	17.687	2.4805	138.91	3.5091
	FSDT [1]	15.834	1.6927	124.36	3.5264
	CBT	15.831	1.0081	124.36	3.5264
20	Present	130.55	1.3794	512.67	7.0451
	HSDT [5]	130.39	1.3771	512.05	7.0441
	FSDT [1]	126.67	1.1792	497.46	7.0528
	CBT	126.67	1.0081	497.46	7.0528
50	Present	1989.3	1.0677	3124.8	17.6313
	HSDT [5]	1988.6	1.0672	3123.7	17.6285
	FSDT [1]	1979.3	1.0355	3109.1	17.6319
	CBT	1979.3	1.0081	3109.1	17.6319
100	Present	15856	1.0231	12453	35.2678
	HSDT [5]	15853	1.0229	12451	35.2624
	FSDT [1]	15834	1.0149	12436	35.2639
	CBT	15834	1.0081	12436	35.2639

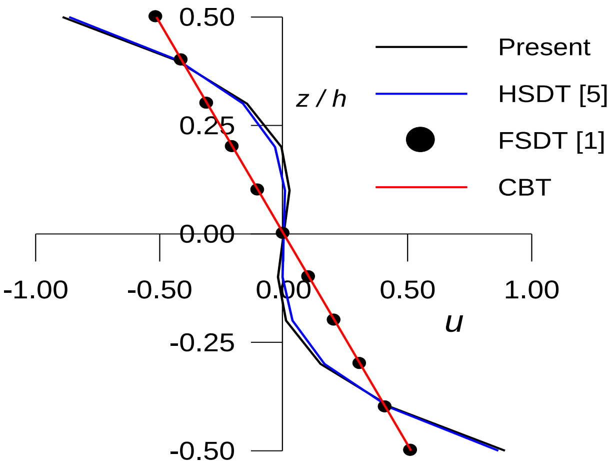

Figure 3. The through thickness variation of the axial displacement for the three-layered (0°/90°/0°) laminated composite beam subjected to the sinusoidal load

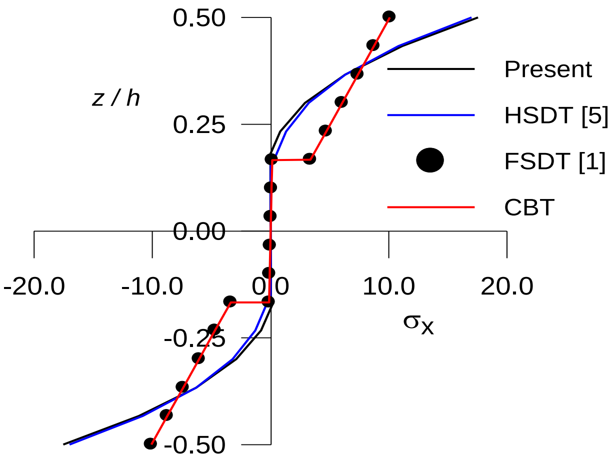

Figure 4. The through thickness variation of the normal stress for the three-layered (0°/90°/0°) laminated composite beam subjected to the sinusoidal load

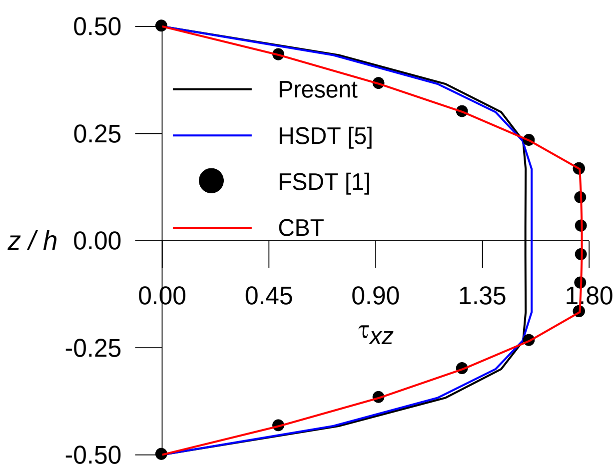

Figure 5. The through thickness variation of the transverse shear stress for the three-layered (0°/90°/0°) laminated composite beam subjected to the sinusoidal load

It is observed from the results that the present theory is in excellent agreement with HSDT to predict the bending response of soft core sandwich beams. The through thickness distributions of displacement and stresses are shown in Figs. 6-8.

Example 3:

In this example, the bending response of the five-layered (0°/90°/core/90°/0°) soft core sandwich beams is investigated as shown in Fig. 2 (c). The beam has two face sheets at the top and bottom and transversely flexible core at the center. The thickness of each face sheet is $0.05h$ each and thickness of core is $0.8h$. The comparison of displacements and stresses for the beam subjected to the sinusoidal load is shown in Table 4 for various aspect ratios (L/h).

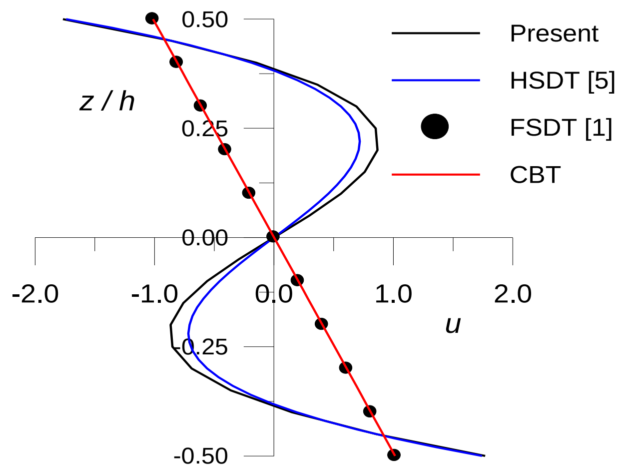

Figure 6. The through thickness variation of the axial displacement for the three-layered (0°/core/0°) soft core sandwich beam subjected to the sinusoidal load

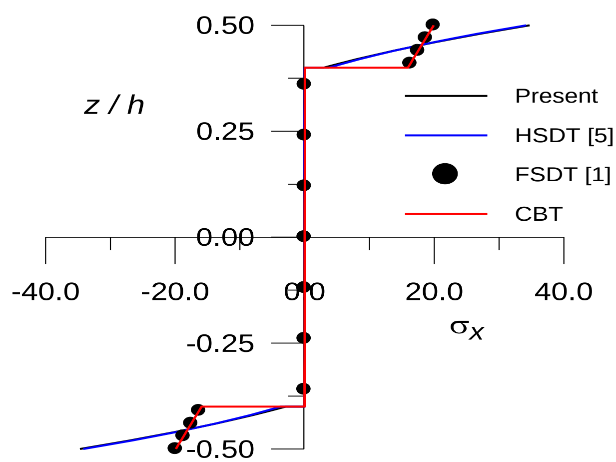

Figure 7. The through thickness variation of the normal stress for the three-layered (0°/core/0°) soft core sandwich beam subjected to the sinusoidal load

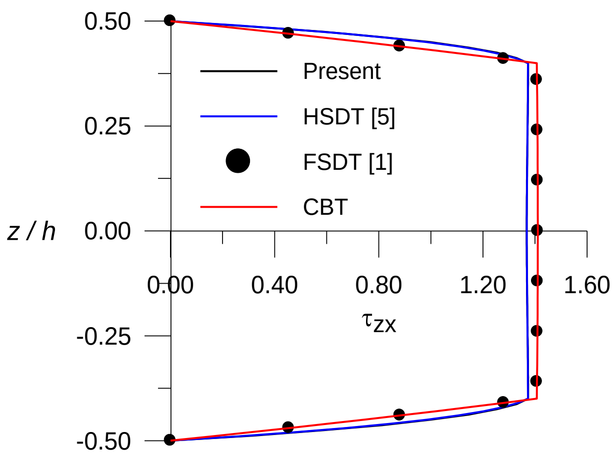

Figure 8. The through thickness variation of the transverse shear stress for the three-layered (0°/core/0°) soft core sandwich beam subjected to the sinusoidal load

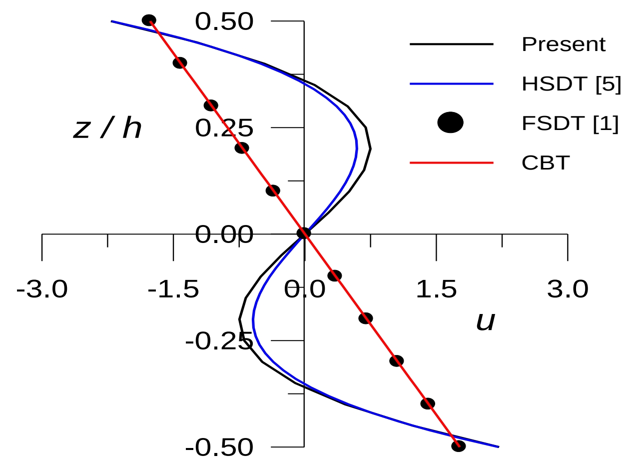

Figure 9. The through thickness variation of the axial displacement for the five-layered (0°/90°/core/90°/0°) soft core sandwich beam subjected to the sinusoidal load

Table 4: The comparison of the axial displacement ($\bar{u}$), the transverse displacement ($\bar{w}$), the normal stress ($\bar{\sigma}_x$), and the transverse shear stress ($\bar{\tau}_{zx}$), for five-layered (0°/90°/core/90°/0°) soft core sandwich beam subjected to the sinusoidal load

L/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}$
4	Present	2.2146	10.929	43.484	1.3402
	HSDT [5]	2.2070	10.925	43.334	1.3376
	FSDT [1]	1.7660	7.1233	34.676	1.3469
	CBT	1.7660	1.7567	34.676	1.3469
10	Present	28.718	3.2141	225.55	3.3630
	HSDT [5]	28.702	3.2315	225.42	3.3623
	FSDT [1]	27.594	2.6154	216.73	3.3674
	CBT	27.594	1.7567	216.73	3.3674
20	Present	223.00	2.1212	875.74	6.7326
	HSDT [5]	222.97	2.1257	875.62	6.7322
	FSDT [1]	220.75	1.9714	866.92	6.7322
	CBT	220.75	1.7567	866.92	6.7347
50	Present	3454.9	1.8151	5427.0	16.8359
	HSDT [5]	3454.9	1.8150	5426.9	16.8359
	FSDT [1]	3449.3	1.7911	5418.2	16.8368
	CBT	3449.3	1.7567	5418.2	16.8368
100	Present	27606	1.7713	21681	33.6734
	HSDT [5]	27606	1.7714	21681	33.6734
	FSDT [1]	27594	1.7653	21673	33.6735
	CBT	27594	1.7567	21673	33.6735

The results show that the axial displacement and stresses are increased with an increase in the aspect ratio, while the transverse displacement is decreased. Since an exact solution for this example is not available in the literature, the results of the present theory are compared with other theories and are found to agree well with each other. Table 5 shows the displacements and stresses for the five-layered soft core sandwich beams subjected to the uniform load. The through thickness distributions of the axial displacement, the normal stress and the transverse shear stress via equilibrium equation are shown in Figs. 9-11.

Table 5: The comparison of the axial displacement ($\bar{u}$), the transverse displacement ($\bar{w}$), the normal stress ($\bar{\sigma}_x$), and the transverse shear stress ($\bar{\tau}_{zx}$), for five-layered (0°/90°/core/90°/0°) soft core sandwich beam subjected to the uniform load

L/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}$
4	Present	2.9647	13.449	51.694	2.2517
	HSDT [5]	2.9438	13.556	51.532	2.1906
	FSDT [1]	2.2816	8.8491	42.782	2.0828
	CBT	2.2816	2.2282	42.782	2.0828
10	Present	37.382	4.0263	276.38	5.1818
	HSDT [5]	37.350	4.0479	276.22	5.1653
	FSDT [1]	35.650	3.2875	267.38	5.2070
	CBT	35.650	2.2282	267.38	5.2070
20	Present	288.67	2.6778	1078.58	10.378
	HSDT [5]	288.60	2.6834	1078.44	10.375
	FSDT [1]	285.20	2.4930	1069.55	10.414
	CBT	285.20	2.2282	1069.55	10.414
50	Present	4464.9	2.3001	6693.68	26.020
	HSDT [5]	4464.8	2.3010	6693.67	26.019
	FSDT [1]	4456.2	2.2705	6684.60	26.035
	CBT	4456.2	2.2282	6684.60	26.035
100	Present	35667.0	2.2464	26747.8	52.062
	HSDT [5]	35667.0	2.2464	26747.8	52.062
	FSDT [1]	35650.0	2.2387	26738.6	52.069
	CBT	35650.0	2.2282	26738.8	52.070

6. Conclusions

In this study, a trigonometric shear deformation theory has been presented for the bending analysis of the soft core sandwich beams. The theory is a displacement-based theory which includes the transverse shear deformation effect. The number of unknown variables is the same as that of the first-order shear deformation theory. The theory satisfies zero shear stress conditions on the top and bottom surfaces of the beam perfectly. Hence, the theory obviates the need for the shear correction factor.

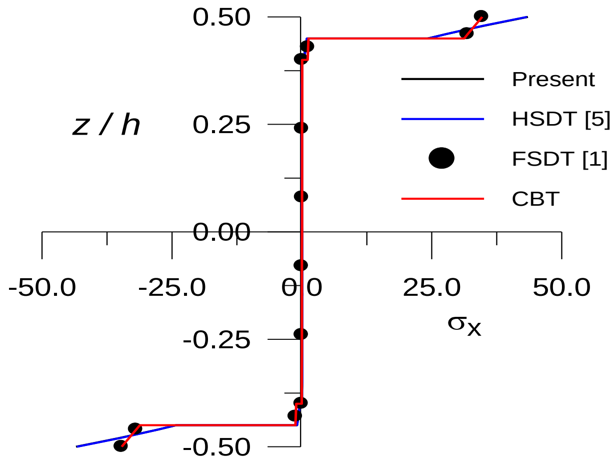

Figure 10. The through thickness variation of the normal stress for the five-layered (0°/90°/core/90°/0°) soft core sandwich beam subjected to the sinusoidal load

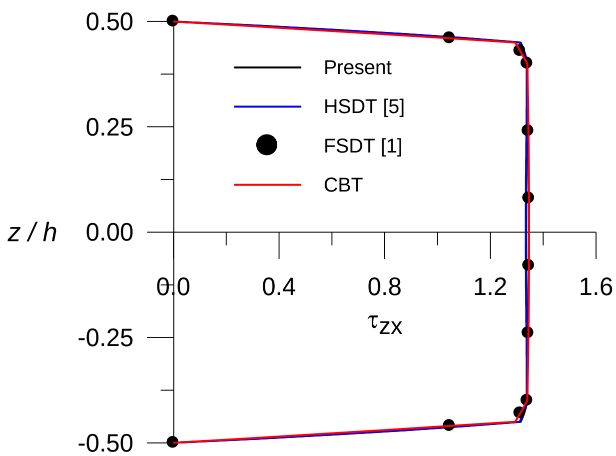

Figure 11. The through thickness variation of the transverse shear stress for the five-layered (0°/90°/core/90°/0°) soft core sandwich beam subjected to the sinusoidal load

From the numerical study and discussion it is concluded that the present theory is in an excellent agreement with other theories, while predicting the bending response of the laminated composite and soft core sandwich beams with transversely flexible core.

References

[1] Timoshenko SP. On the Correction for Shear of the Differential Equation for Transverse Vibrations of Prismatic Bars. *Philos Mag* 1921; 41(6): 742–746.

[2] Lo KH, Christensen RM, Wu EM. A High-order Theory of Plate Deformation, Part-1: Homogeneous Plates, *ASME J Appl Mech* 1977; 44: 663–668.

[3] Lo KH, Christensen RM, Wu EM. A High-order Theory of Plate Deformation, Part-2: Laminated Plates, *ASME J Appl Mech* 1977; 44: 669–676.

[4] Levinson M. A New Rectangular Beam Theory. *J Sound Vib* 1981; 74: 81–87.

[5] Reddy JN. A Simple Higher Order Theory for Laminated Composite Plates. *ASME J Appl Mech* 1984; 51: 745–752.

[6] Touratier M. An Efficient Standard Plate Theory. *Int J Eng Sci* 1991; 29(8): 901–916.

[7] Soldatos KP. A Transverse Shear Deformation Theory for Homogeneous Monoclinic Plates. *Acta Mech* 1992; 94: 195–200.

[8] Karama M, Afaq KS, Mistou S. A Refinement of Ambartsumian Multi-layer Beam Theory. *Comput Struct* 2008; 86: 839–849.

[9] Sayyad AS. Comparison of Various Refined Beam Theories for the Bending and Free Vibration Analysis of Thick Beams. *Appl Comput Mech* 2011; 5: 217–230.

[10] Sayyad AS., Ghugal YM, Borkar RR. Flexural Analysis of Fibrous Composite Beams under Various Mechanical Loadings using Refined Shear Deformation Theories. *Compos Mech Comput Appl* 2014; 5(1): 1–19.

[11] Sayyad AS. Static Flexure and Free Vibration Analysis of Thick Isotropic Beams using Different Higher Order Shear Deformation Theories. *Int J Appl Math Mech* 2012; 8(14): 71–87.

[12] Mechab I, Tounsi A, Benatta MA, Bedia EAA. Deformation of Short Composite Beam using Refined Theories. *J Math Anal Appl* 2008; 346: 468–479.

[13] Carrera E, Giunta G. Refined Beam Theories based on A Unified Formulation, *Int J Appl Mech* 2010; 2(1): 117–143.

[14] Carrera E, Filippi M, Zappino E. Laminated Beam Analysis by Polynomial, Trigonometric, Exponential and Zig-zag Theories. *Eur J Mech A Solids* 2013; 41: 58–69.

[15] Carrera E, Filippi M, Zappino E. Free Vibration Analysis of Laminated Beam by Polynomial, Trigonometric, Exponential and Zig-zag Theories. *J Compos Mater* 2014; 48(19): 2299–2316.

[16] Giunta G, Metla N, Belouettar S, Ferreira AJM, Carrera E. A Thermo-Mechanical Analysis of Isotropic and Composite Beams via Collocation with Radial Basis Functions. *J Therm Stresses* 2013; 36: 1169–1199.

[17] Chakrabarti A, Chalak HD, Iqbal MA, Sheikh AH. A New FE Model based on Higher Order Zigzag Theory for the Analysis of Laminated Sandwich Beam with Soft Core. *Compos Struct* 2011; 93: 271–279.

[18] Chalak HD, Chakrabarti A, Iqbal MA, Sheikh AH. Vibration of Laminated Sandwich Beams Having Soft Core, *J Vib Control* 2011; 18(10): 1422–1435.

[19] Gherlone M, Tessler A, Sciuva MD. A C^0 Beam Elements based on the Refined Zig-zag Theory for Multi-layered Composite and Sandwich Laminates. *Compos Struct* 2011; 93: 2882–2894.
[20] Shimpi RP, Ghugal YM. A New Layerwise Trigonometric Shear Deformation Theory for Two-layered Cross-ply Beams. *Compos Sci Technol* 2001; 61: 1271–1283.
[21] Ghugal YM, Shinde SB. Flexural Analysis of Cross-ply Laminated Beams using Layerwise Trigonometric Shear Deformation Theory. *Latin Am J Solids Struct* 2013; 10(4): 675–705.
[22] Arya H. A New Zig-zag Model for Laminated Composite Beams: Free Vibration Analysis. *J Sound Vib* 2003; 264: 485–490.
[23] Sayyad AS, Ghugal YM. Effect of Transverse Shear and Transverse Normal Strain on Bending Analysis of Cross-ply Laminated Beams. *Int J Appl Math Mech* 2011; **7**(12): 85–118.
[24] Mantari JL, Oktem AS, Soares CG. A New Trigonometric Shear Deformation Theory for Isotropic, Laminated Composite and Sandwich Plates. *Int J Solids Struct* 2012; 49(1): 43–53.
[25] Ferreira AJM, Roque CMC, Jorge RMN. Analysis of Composite Plates by Trigonometric Shear Deformation Theory and Multi-quadrics. *Comput Struct* 2005; 83(27): 2225–2237.
[26] Zenkour AM. Benchmark Trigonometric and 3-D Elasticity Solutions for an Exponentially Graded Thick Rectangular Plate. *Arch Appl Mech* 2007; 77: 197–214.
[27] Sayyad AS, Ghugal YM, Naik NS. Bending Analysis of Laminated Composite and Sandwich Beams according to Refined Trigonometric Beam Theory. *Curved and Layered Struct* 2015; 2: 279–289.
[28] Dahake AG, Ghugal YM. A Trigonometric Shear Deformation Theory for Flexure of Thick Beams. *Procedia Eng* 2013; 51: 1–7.
[29] Dahake AG, Ghugal YM. A Trigonometric Shear Deformation Theory for Flexure of Thick Beams. *Int J Sci Res Publ* 2012; 2(11): 1–7.
[30] Ghugal YM, Dahake AG. Flexure of Cantilever Thick Beams using Trigonometric Shear Deformation Theory. *Int J Mech Aerosp Ind Mechatronic Manuf Eng* 2013; 7(5): 380–389.
[31] Pagano NJ. Exact Solutions for Composite Laminates in Cylindrical Bending. *Compos Mater* 1969; 3: 398–411.