



Semnan University

Mechanics of Advanced Composite Structures

journal homepage: <http://MACS.journals.semnan.ac.ir>

An Analytical Study on Effects of adding Nanoparticles to Water and Enhancement in Thermal Properties Based on Falkner-Skan Model

Y. Rostamiyan^{a*}, M. Abbasi^a, F. Aghajani^a, F. Hedayati^a, S.M. Hamidi^b

^a Department of Mechanical Engineering, Sari Branch, Islamic Azad University, Sari, Iran

^b Department of Electrical and Energy Engineering, University of Vaasa, Vaasa 65200, Finland

PAPER INFO

Paper history:

Received 6 August 2014

Received in revised form 15

September 2014

Accepted 22 September 2014

Keywords:

Falkner-skan

Boundary layer

Nanofluids

HAM

ABSTRACT

In the age of technology, it is vital to cool down different parts of a device to use it more beneficially. Using nanofluids is one of the most common methods which has shown very effective results. In this paper, we have rephrased a classic equation in fluid mechanics, i.e. the Falkner-Skan boundary layer equation, in order to be used for nanofluid. This nonlinear equation, which was presented by Liao, has been solved by Homotopy Analysis Method (HAM). This method is very capable to solve a wide range of nonlinear equations. The physical interpretation of results which are velocity and temperature profiles are explained in details and they are parallel with experimental outcomes of previous researchers.

© 2014 Published by Semnan University Press. All rights reserved.

1. Introduction

As technology improves, it was realized that devices have to cool down in a more effective way and the conventional fluids such as water are not appropriate anymore, so the idea of adding particles to a fluid was presented. These tiny particles have high thermal conductivity, so the mixed fluids have better thermal properties [1-3]. The material of these nano scale particles is aluminum oxide (Al_2O_3), copper (Cu), copper oxide (CuO), gold (Au), silver (Ag) etc, which are suspending in base fluids such as water, oil, acetone and ethylene glycol, etc. Al_2O_3 and CuO are the most well-known nanoparticles used by many researchers [4-8] in their experimental researches. They claimed different results due to the size and shape and so the contact surface of the particles. In addition the base fluid characteristics were important as well. The main obstacle in this field was how to keep the particles suspended in static fluid

which is discussed in [5-9]. Fortunately, the result was in a same trace that the thermal conductivity of the nanofluids is higher than the conventional fluids and this term is modeled mathematically in [4, 5], [10-15]. In this paper we will rephrase the Falkner-Skan equation [16-18] for a nanofluid with a semi analytical method, i. e. Homotopy Analysis Method, which was presented by Liao [19-21] in 1992. The HAM is one of the well-known methods to solve nonlinear equations. This method has been used by many authors in a wide range of engineering problems [22-25]. Falkner and Skan considered two-dimensional wedge flows. They developed a similarity solution method in which the partial differential boundary-layer equation was reduced to a nonlinear third-order ordinary differential equation which does not have an exact solution ;besides, the numerical solution for this equation is time consuming and difficult.

* Corresponding author.

2. Governing Equations

In this paper, we consider an incompressible viscous fluid which flows over a wedge, as is shown in Fig. 1.

The wall temperature, i.e. T_w , is uniform and constant and is greater than the free stream temperature, T_1 . It is assumed that the free stream velocity, U_1 , is also uniform and constant as well. Further, assuming that the flow in the laminar boundary layer is two-dimensional, and that the temperature gradients resulting from viscous dissipation are small, the continuity, momentum and energy equations can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v are the respective velocity components in the x and y direction of the fluid flow, ν is the kinematic viscosity of the fluid, and U is the reference velocity at the edge of the boundary layer and is a function of x . α is the thermal diffusivity of the fluid, T is the temperature in the vicinity of the wedge, and the boundary conditions are given by:

$$\text{at } y = 0 : u = v = 0 \text{ and } T = T_w \quad (4)$$

$$\lim_{y \rightarrow \infty} u = U(x) = U_\infty \left(\frac{x}{L}\right)^m \text{ and } \lim_{y \rightarrow \infty} T = T_\infty \quad (5)$$

$$\text{at } x = 0 : u = u_\infty \text{ and } T = T_\infty \quad (6)$$

where U_∞ is the mean stream velocity, L is the length of the wedge, m is the Falkner-Skan power-law parameter, and x is measured from the tip of the wedge. A stream function, $\psi(x, y)$, is introduced

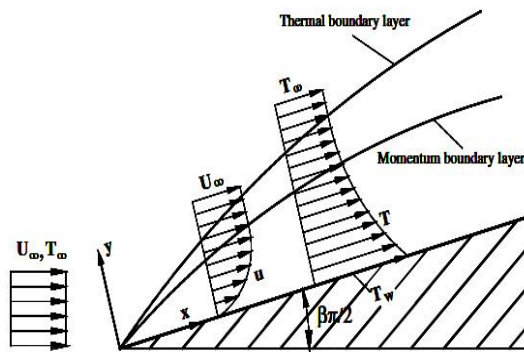


Figure 1. Velocity and thermal boundary layers for the Falkner-Skan wedge flow.

such as this:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (7)$$

By Substituting Eq. (7) into Eq. (2), the momentum equation will be obtained as follows:

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = U \frac{dU}{dx} + \nu \frac{\partial^3 \psi}{\partial x^3} \quad (8)$$

By using these similarity variable yields, we obtain the following ordinary differential equation:

$$f(\eta) = \sqrt{\left(\frac{m+1}{2}\right) \left(\frac{L^m}{\nu_f u_\infty}\right)} \frac{\psi}{x^{\frac{m+1}{2}}} \quad (9)$$

$$\eta = \sqrt{\frac{m+1}{2} \frac{\nu u_\infty}{L^m} \frac{y}{x^{\frac{m+1}{2}}}} \quad (10)$$

in which ν_f is the kinematic viscosity of the fluid. Substituting Eqs. (9) and (10) into Eq. (8), gives:

$$\frac{\partial^3 f(\eta)}{\partial \eta^3} + f(\eta) \times \frac{\partial^2 f(\eta)}{\partial \eta^2} + \left(\frac{2m}{m+1}\right) \times \left(1 - \left(\frac{df(\eta)}{\eta}\right)^2\right) = 0 \quad (11)$$

this is known as the Falkner-Skan boundary-layer equation [13-15]. The boundary conditions of $f(\eta)$ are:

$$\text{at } \eta = 0 : f(0) = \frac{df(\eta)}{\eta} = 0 \quad (12)$$

$$\lim_{\eta \rightarrow \infty} \frac{df(\infty)}{d\eta} = 1 \quad (13)$$

Note that in the equations above, parameters β and m are related through the expression. β is the Hartree pressure gradient parameter which corresponds to $\beta = \Omega/\pi$ for a total angle Ω of the wedge and $m=0(\beta=0)$ represents the boundary layer flow past a horizontal flat plate, while $m=1(\beta=1)$ corresponds to the boundary layer flow near the stagnation point of a vertical flat plate. m is a dimensionless constant. In the Blasius solution, $m = 0$ corresponds to an angle of attack of zero radians. Thus we can write:

$$\beta = \frac{2m}{m+1}$$

A dimensionless temperature is defined as follows:

$$\theta = \frac{T - T_w}{T_\infty - T_w} \quad (14)$$

If Eq. (14) is substituted into Eq. (3), then the boundary-layer energy equation becomes:

$$\frac{\partial^2 \theta(\eta)}{\partial \eta^2} + Pr f(\eta) \frac{df(\eta)}{\eta} = 0 \tag{15}$$

where $Pr = \nu / \alpha$ is the Prandtl number. With the following boundary conditions:

$$\text{at } \eta = 0 : \theta(0) = 1 \tag{16}$$

$$\lim_{\eta \rightarrow \infty} \theta(\eta) = \theta(\infty) = 1 \tag{17}$$

The following equations can be obtained by changing these parameters to rephrase this classic equation and use it for nanofluids:

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \tag{18}$$

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_p \tag{19}$$

$$\psi = \sqrt{\frac{2\nu_f x u_e(x)}{m + 1}} f(\eta) \tag{20}$$

$$\eta = \sqrt{\frac{(m + 1)u_e(x)}{2\nu_f x}} y \tag{21}$$

$$(\rho c_p)_{nf} = (1 - \varphi)(\rho c_p)_f + (\varphi \rho c_p)_p \tag{22}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_p + 2k_f) - 2\varphi(k_f - k_p)}{(k_p + 2k_f) + \varphi(k_f - k_p)} \tag{23}$$

Finally:

$$\frac{1}{(1 - \varphi)^{2.5}(1 - \varphi + \varphi \frac{\rho_s}{\rho_f})} f''' + f f'' + \left(\frac{2m}{m + 1}\right) (1 - f'^2) = 0 \tag{24}$$

With these boundary conditions:

$$\text{at } \eta = 0 : f(0) = \frac{df(\eta)}{d\eta} = 0 \tag{25}$$

$$\lim_{\eta \rightarrow \infty} \frac{df(\infty)}{d\eta} = 1 \tag{26}$$

And also:

$$\frac{1}{Pr} \frac{\left(\frac{k_{nf}}{k_f}\right)}{(1 - \varphi + \varphi \frac{(\rho c_p)_s}{(\rho c_p)_f})} \theta'' + f \theta' = 0 \tag{27}$$

With these boundary conditions:

$$\text{at } \eta = 0: \theta(0) = 0 \tag{28}$$

$$\lim_{\eta \rightarrow \infty} \theta(\eta) = \theta(\infty) = 1 \tag{29}$$

The physical properties are also mentioned in the Table 1.

3. Basic Idea of HAM

Let us assume the following nonlinear differential equation in the form of:

$$N[u(\tau)] = 0 \tag{30}$$

where N is a nonlinear operator, τ is an independent variable and $u(\tau)$ is the solution of equation. We define the function, $\varphi(\tau, p)$ as follows:

$$\lim_{p \rightarrow 0} \varphi(\tau, p) = u_0(\tau) \tag{31}$$

where, $p \in [0,1]$ and $u_0(\tau)$ is the initial guess which satisfies the initial or boundary conditions and if

$$\lim_{p \rightarrow 1} \varphi(\tau, p) = u(\tau) \tag{32}$$

And by using the generalized homotopy method, Liao's so-called zero-order deformation Eq. (30) will be:

$$(1 - P) L [\varphi(\tau, p) - u_0(\tau)] = p \hbar H(\tau) N[\varphi(\tau, p)] \tag{33}$$

where \hbar is the auxiliary parameter which helps us increase the results convergence, $H(\tau)$ is the auxiliary function and L is the linear operator. It should be noted that there is a great freedom to choose the auxiliary parameter \hbar , the auxiliary function $H(\tau)$, the initial guess $u_0(\tau)$ and the auxiliary linear operator L .

Thus, when p increases from 0 to 1 the solution $\varphi(\tau, p)$ changes between the initial guess $u_0(\tau)$ and the solution $u(\tau)$. The Taylor series expansion of $\varphi(\tau, p)$ with respect to p is:

$$\varphi(\tau, p) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau) p^m \tag{34}$$

And

$$u_0^{[m]}(\tau) = \frac{\partial^m (\varphi(\tau; p))}{\partial p^m} \Big|_{p=0} \tag{35}$$

where $u_0^{[m]}(\tau)$ for brevity is called the m^{th} order of

Table 1. Thermophysical properties of the base fluid and the nanoparticles (Oztop and Abu-Nada [36]).

Physical properties	Fluid phase (water)	Cu	Al ₂ O ₃
c_p (J/kg K)	4179	385	765
ρ (kg /m ³)	997.1	8933	3970
k (W/m K)	0.613	400	40
$\alpha \times 10^{-7}$ (m ² /s)	1.47	1163.1	131.7

deformation derivation which reads:

$$u_m(\tau) = \frac{u_0^{[m]}}{m!} = \frac{1}{m!} \frac{\partial^m (\varphi(\tau; p))}{\partial p^m} \Big|_{p=0} \quad (36)$$

According to the definition in Eq. (36), the governing equation and the corresponding initial conditions of $u_m(\tau)$ can be deduced from zero-order deformation Eq. (30). Differentiating Eq. (30) for m times with respect to the embedding parameter p and setting $p = 0$ and finally dividing by $m!$, we will have the so-called m^{th} order deformation equation in the following form:

$$L[u_m(\tau) - \chi_m u_{m-1}(\tau)] = \hbar H(\tau) R(u_{m-1}^{\rightarrow}) \quad (37)$$

where:

$$R(u_{m-1}^{\rightarrow}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\varphi(\tau, p)]}{\partial p^{m-1}} \quad (38)$$

And

$$\chi_m = \begin{cases} 0 & m \leq 1 \\ 1 & m > 1 \end{cases} \quad (39)$$

So by applying an inverse linear operator to both sides of the linear equation, Eq. (30), we can easily solve the equation and compute the generation constant by applying the initial or boundary condition.

4. Application of HAM to Falkner-Skan Problem

As mentioned by Liao [19-21], a solution may be expressed with different base functions, among which some converge to the exact solution of the problem faster than others. Such base functions are obviously better suited for the final solution to be expressed. Noting these facts, we have decided to express $f(\eta)$ and $\theta(\eta)$ by a set of base functions in the following form:

$$f(\eta) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta) \quad (40)$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta) \quad (41)$$

The rule of solution expression provides us with a starting point. It is under the rule of solution expression that the initial approximations, the auxiliary linear operators, and the auxiliary functions are determined. So, according to the rule of solution expression, we choose the initial guess and auxiliary linear operator in the following form

$$f_0(\eta) = \exp(-\eta) + \eta - 1 \quad (42)$$

$$\theta_0(\eta) = \exp(-\eta) \quad (43)$$

$$L_1(f) = f''' + f'' \quad (44)$$

$$L_2(\theta) = \theta'' + \theta' \quad (45)$$

$$L_1(C_1 + C_1\eta + C_3 \exp(-\eta)) = 0 \quad (46)$$

$$L_2(c_4 + c_5 \exp(-\eta)) = 0 \quad (47)$$

where $c_i (i = 1_5)$ are constants. Let $p \in [0,1]$ denotes the embedding parameter and \hbar indicates non-zero auxiliary parameters. We construct the following equations:

4.1 Zeroth-Order Deformation Equations

$$(1-P)L_1[f(\eta; p) - f_0(\eta)] = p \hbar N_1[f(\eta; p), \theta(\eta; p)] \quad (48)$$

$$f(0; p) = 0; f'(0; p) = 0; f(\infty; p) = 1 \quad (49)$$

$$(1-P)L_2[\theta(\eta; p) - \theta_0(\eta)] = p \hbar N_2[f(\eta; p), \theta(\eta; p)] \quad (50)$$

$$\theta(0; p) = 1; \theta(\infty; p) = 0 \quad (51)$$

$$N_1[F(\eta; p), G(\eta; p)] = \frac{1}{(1-\varphi)^{2.5} \left(1 - \varphi + \varphi \frac{\rho_s}{\rho_f}\right)} \frac{\partial^3 f(\eta; p)}{\partial \eta^3} + f(\eta; p) \frac{\partial^2 f(\eta; p)}{\partial \eta^2} + \left(\frac{2m}{m+1}\right) \left(1 - \left(\frac{\partial f(\eta; p)}{\partial \eta}\right)^2\right) \quad (52)$$

$$N_2[F(\eta; p), G(\eta; p)] = \frac{1}{Pr \left(1 - \varphi + \varphi \frac{(\rho C_p)_s}{(\rho C_p)_f}\right)} \frac{\partial^2 \theta(\eta; p)}{\partial \eta^2} + f(\eta; p) \frac{\partial \theta(\eta; p)}{\partial \eta} \quad (53)$$

For $p = 0$ and $p = 1$ we have:

$$f(\eta; 0) = f_0(\eta) f(\eta; 1) = f(\eta) \quad (54)$$

$$\theta(\eta; 0) = \theta_0(\eta) \theta(\eta; 1) = \theta(\eta) \quad (55)$$

When p increases from 0 to 1 then $f(\eta; p)$ and $\theta(\eta; p)$ vary from $f_0(\eta)$ and $\theta_0(\eta)$ to $f(\eta)$ and $\theta(\eta)$. Using Taylor's theorem for Eqs. (54) and (55), $f(\eta; p)$ and $\theta(\eta; p)$ can be expanded in a power series of p as follows:

$$f(\eta; 0) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, f_m(\eta) = \frac{1}{m!} \frac{\partial^m(f(\eta; p))}{\partial p^m} \tag{56}$$

$$\theta(\eta; 0) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m, \theta_m(\eta) = \frac{1}{m!} \frac{\partial^m(\theta(\eta; p))}{\partial p^m} \tag{57}$$

in which \hbar is chosen in such a way that these two series are convergent at $p = 1$, therefore we have through Eqs. (56) and (57) that

$$f(\eta; 0) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \quad \theta(\eta; 0) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{58}$$

4.2 *m*th -order deformation equations

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 H_f(\eta) R_m^f(\eta) \tag{59}$$

$$f_m(0) = 0; f'_m(0) = 0; f'_m(\infty) = 0 \tag{60}$$

$$L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_2 H_\theta(\eta) R_m^\theta(\eta) \tag{61}$$

$$\theta_m(0) = 0; \theta_m(\infty) = 0 \tag{62}$$

$$R_m^f(\eta) = \frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\frac{\rho_s}{\rho_f})} f'''_{m-1} + \sum_{n=0}^{m-1} f_{m-1-n} f''_n \tag{63}$$

$$R_m^\theta(\eta) = \frac{1}{Pr} \frac{\frac{k_{nf}}{k_f}}{(1-\varphi+\varphi\frac{(\rho C_p)_s}{(\rho C_p)_f})} \theta''_{m-1} + \sum_{n=0}^{m-1} f_{m-1-n} \theta'_n \tag{64}$$

$$\chi_m = \begin{cases} 1, & m \leq 1 \\ 0, & m > 1 \end{cases} \tag{65}$$

The general solutions will be:

$$f_m(\eta) - \chi_m f_{m-1}(\eta) = f_m^*(\eta) + (C_1^m + C_2^m \eta) + C_3^m \exp(-\eta) \tag{66}$$

$$\theta_m(\eta) - \chi_m \theta_{m-1}(\eta) = \theta_m^*(\eta) + (C_4^m + C_5^m \exp(-\eta)) \tag{67}$$

where C_1^m to C_5^m are constants that can be obtained by applying the boundary conditions in Eqs. (60) and (62).

As discussed by Liao the rule of coefficient ergodicity and the rule of solution existence play important roles in determining the auxiliary function and ensuring that the high-order deformation equations are closed and have solutions. In many cases, by means of the rule of solution expression and the rule of coefficient ergodicity, auxiliary functions can be uniquely determined. So we define the auxiliary functions $H_f(\eta)$ and $H_\theta(\eta)$ in the following form:

$$H_f(\eta) = 1 \tag{68}$$

$$H_\theta(\eta) = \exp(-\eta) \tag{69}$$

5. Convergence of HAM Solution

HAM provides us with great freedom in choosing the solution of a nonlinear problem by different base functions. This has a great effect on the convergence region because the convergence region and the rate of a series are chiefly determined by the base functions used to express the solution. Therefore, a more accurate approximation of a nonlinear problem can be obtained by choosing a proper set of base functions and ensuring about its convergency.

On the other hand, as pointed out by Liao, the convergence and the rate of approximation for the HAM solution strongly depend on the value of auxiliary parameters \hbar . Even if the initial approximations $f_0(\eta)$ and $\theta_0(\eta)$, the auxiliary linear operator L , and the auxiliary functions $H_f(\eta)$ and $H_\theta(\eta)$ are given, we still have great freedom to choose the value of the auxiliary parameters \hbar_1 and \hbar_2 . So, the auxiliary parameters provide us with an additional way to conveniently adjust and control the convergence region and the rate of solution series. By means of the so-called \hbar -curves it is easy to find out the so-called valid regions of auxiliary parameters to gain a convergent solution series. When the valid region of auxiliary parameters is a horizontal line segment, then the solution is converged.

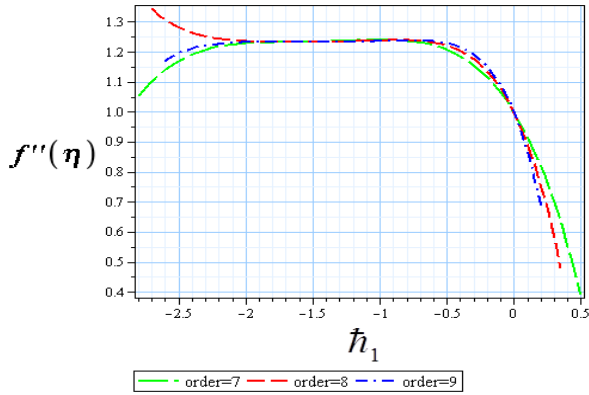


Figure 2. The \hat{h}_1 -validity for CuO for $m = 1$.

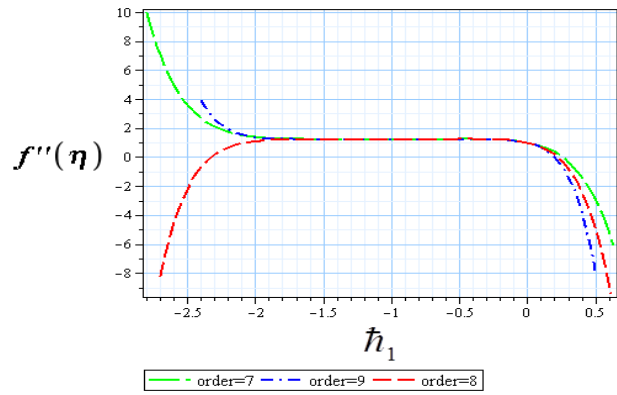


Figure 4. The \hat{h}_1 -validity for Al₂O₃ for $m = 1$.

In our case study, suitable range of \hat{h}_1 and \hat{h}_2 for CuO and Al₂O₃ can be obtained from Figures 2 to 5.

6. Results and Discussion

Fig. 6 shows the relation between $f'(\eta)$ and η which represents the velocity of the fluid. Adding nano particles to a base fluid will cause a rise in fluid viscosity, therefore, the momentum exchange between the nanofluid layers is higher. It can be clearly seen that the pure fluid reaches to 1 approximately in $\eta = 5$, however, nanofluids approach to 1 in $\eta = 9$. Considering Eq. (18), we have expected this trend. Fig. 7 shows temprature curve variation with φ which represents the particle volume fraction of the suspension. When φ climbs up from 0.3 % to 0.9 %, as we expected, there is a noticeable enhancement in heat transfer rate. This can be justified by torbulace which the nano-scale particles cause. In addition, with the increase in the amount of φ the effects of the nanoparticles conductivity appear in the heat transfer rate as well.

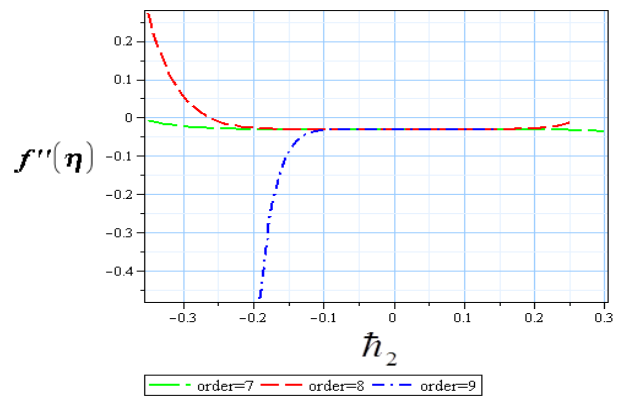


Figure 5. The \hat{h}_2 -validity for Al₂O₃ for $m = 1$.

As we can see at about $\eta = 4.5$ the nanofluid curve with $\varphi = 0.9\%$, has dropped to 0 but when $\varphi = 0.3\%$ this happens at $\eta = 7$.

This trend was observed before in experimental studies by others [3].

Fig. 8 shows the bulk temperature as a function of the fluids types.

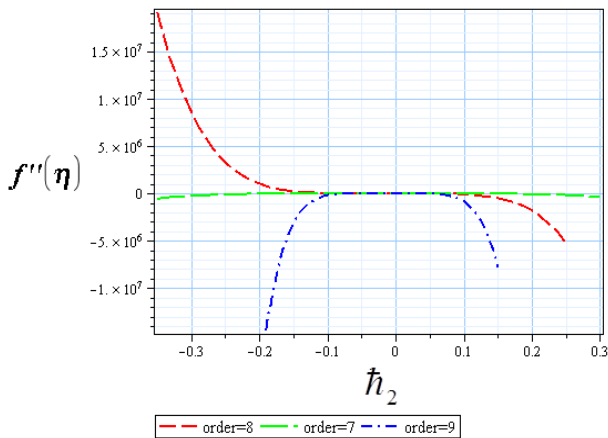


Figure 3. The \hat{h}_2 -validity for CuO for $m = 1$.

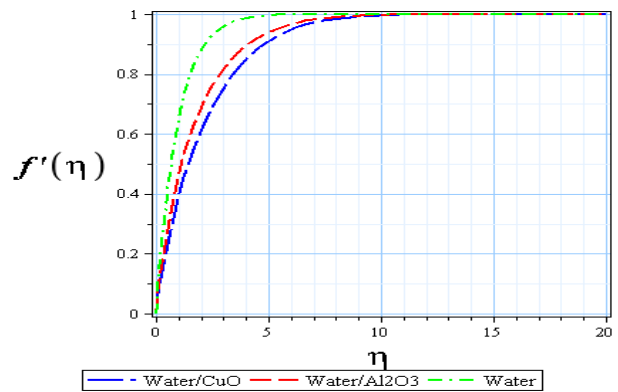


Figure 6. Velocity profile for different types of fluids for $m = 1$.

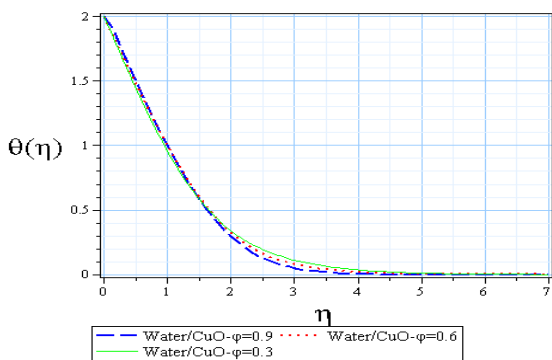


Figure 7. Temperature profile for CuO for different values of ϕ and $m = 1$.

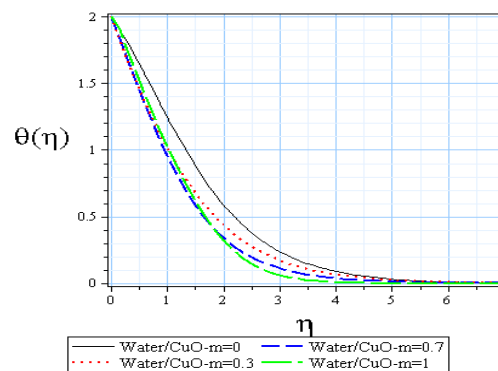


Figure 9. Temperature profile for CuO for different values of m and $\phi = 0.1\%$.

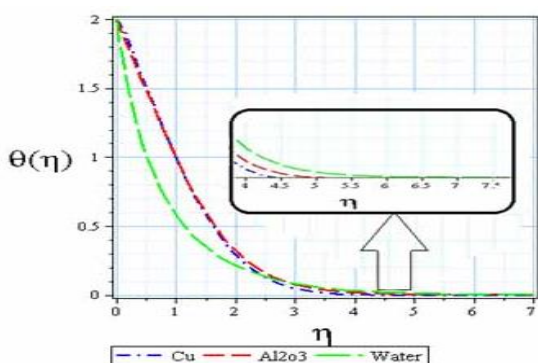


Figure 8. Temperature profile for different types of fluids for $\phi = 0.1\%$ and $m = 1$

In agreement with the previous experimental studies results, our solution shows that nanofluids have better thermal properties and the temperature difference between the fluid and the wedge disappears sooner.

Due to adding the nano particles in the base fluid, we can see a noticeable enhancement in heat transfer rate.

In other words, the nano fluids temperatures are equal with the wedge surface at 4.4 for “CuO/water” and 5.1 for “Al₂O₃/water”, however, water experiences this at 6.5. To see this effect better, we have magnified the region between $3 < \eta < 8$ in the related figure.

Fig. 9 shows the relation between fluid’s bulk temperature, i.e. $\theta(\eta)$ and m . We can observe a marked change in the η at which profiles drop to 0. As m grows, the effects of the stagnation point will be more important and the conductivity of the fluid will play a more important role. Because of high heat conduction in nanofluids the curve approaches to 0 sooner with the increase in m .

7. Conclusion

To make it brief, in this paper, we derived the Falkner-Skan equation for a nanofluid and solved it with the Homotopy Analysis Method. The convergence region of HAM is discussed, i.e. the auxiliary parameters \hbar_1 and \hbar_2 .

The physical interpretation of the results was explained.

Temperature and velocity diagrams have been discussed in details which has shown that CuO as a nanoparticle has more efficiency in comparison with Al₂O₃. Both of these nanoparticles are suspended in water as the base fluid. In addition we observed that with adding nano-scale particles to a fluid, the viscosity of the fluid will increase and this will affect the momentum exchange in the fluid layers.

References

- [1] Lee S, Choi SUS, Li S, Eastman JA. Measuring thermal conductivity of fluids containing oxidenanoparticles. ASME J Heat Transfer 1999; 121: 280-9.
- [2] Xuan Y, Li Q. Heat transfer enhancement of nanofluids. Int J Heat Fluid Flow 2000;21:58-64.
- [3] Das SK, Putra N, Roetzel W. Pool boiling characteristics of nano-fluids. Int J Heat Mass Transfer 2003; 46: 851-62.
- [4] Lee S, Choi SUS, Li S, Eastman JA. Measuring thermal conductivity of fluids containing oxidenanoparticles. ASME J Heat Transfer 1999; 121: 280-9.
- [5] Xuan Y, Li Q.: Heat transfer enhancement of nanofluids. Int J Heat Fluid Flow 2000; 21: 58-64.
- [6] Das SK, Putra N, Roetzel W. Pool boiling characteristics of nano-fluids. Int J Heat Mass Transfer 2003; 46: 851-62.

- [7] Das SK, Putra N, Roetzel W. Pool boiling of nanofluids on horizontal narrow tubes. *Int J Multiphase Flow* 2003; 29: 1237–47.
- [8] Das SK, Putra N, Thiesen P, Roetzel W. Temperature dependence of thermal conductivity enhancement for nanofluids. *ASME J Heat Transfer* 2003; 125: 567–74.
- [9] Wang X, Xu X, Choi SUS. Thermal conductivity of nanoparticle–fluid mixture. *J Therm Phys Heat Transfer* 1999; 13: 474–80.
- [10] Eastman JA, Choi SUS, Li S, Yu W, Thomson LJ. Anomalous increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Appl Phys Lett* 2001; 78: 718–20.
- [11] Xie H, Wang J, Xi T, Liu Y, Ai F, Wu Q. Thermal conductivity enhancement of suspensions containing nanosized alumina particles. *Appl Phys* 2002; 91: 4568–72.
- [12] Dongsheng Wen, Guiping Lin, SaeidVafaei, Kai Zhang. *Particuology* 2009; 7: 141–150.
- [13] Wang BX, Zhou LP, Peng XF. A fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles. *Int J Heat Mass Transfer* 2003; 46: 2665–72.
- [14] Xue QZ. Model for effective thermal conductivity of nanofluids. *Phys Lett A* 2003; 307: 313–7.
- [15] Yu W, Choi SUS. The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Maxwell model. *Nanoparticle Res* 2003; 5: 167–71.
- [16] Falkner, V. M, and Skan, S. W. Solutions of the boundary-layer equations. *Philosophical Magazine*, 1931; 7(12): 865–896.
- [17] S. Abbasbandy and T. Hayat. Solution of the MHD Falkner-Skan flow by homotopy analysis method, *Commun Nonlinear Sci Numer Simul* 2009; (14): 3591–3598.
- [18] Pade. On the solution of Falkner–Skan equations, *J Math Anal Appl* 2003; (285): 264–274.
- [19] Liao, S. J. On the Homotopy Analysis Method for Nonlinear Problems. *Appl Math Comput* 2004; 47 (2)
- [20] Liao, S. J. An Explicit, Totally Analytic Approximation of Blasius’s Viscous Flow Problems. *Int J Non-Linear Mech* 1999; 34 (4)
- [21] Liao, S. J. Proposed Homotopy Analysis Techniques for the Solution of Nonlinear Problems. Ph.D. Dissertation, Shanghai Liao Tong University, China 1992.
- [22] Domairry, G. and N. Nadim. Assessment of Homotopy Analysis Method and Homotopy Perturbation Method in Non-Linear Heat Transfer Equation. *Int Commun Heat Mass Transfer* 2008; 35 (1):
- [23] Domairry, G. and M. Fazeli. Homotopy Analysis Method to Determine the Fin Efficiency of Convective Straight Fins with Temperature-Dependent Thermal Conductivity. *Commun Non-linear Sci Numer Simul* 2009; 14 (2)
- [24] Domairry, G., A. Mohsenzadeh, and M. Famouri. The Application of Homotopy Analysis Method to Solve Nonlinear Differential Equation Governing Jeffery–Hamel Flow. *Commun Non-linear Sci Numer Simul* 2009; 14 (1): 85
- [25] Fakhari A, Domairry G, Ebrahimpour. Approximate explicit solutions of nonlinear BBMB equations by homotopy analysis method and comparison with the exact solution. *Phys Lett A* 2007; 368: 64–68.
- [26] Sohoulou AR, Domairry D, Famouri M, Mohsenzadeh A. Analytical solution of natural convection of Darcian fluid about a vertical full cone embedded in porous media prescribed wall temperature by means of HAM, *Int Commun Heat and Mass Transfer* 2008; 35(10): 1380–1384.
- [27] Ziabakhsh Z, Domairry G. Analytic solution of natural convection flow of a non-Newtonian fluid between two vertical flat plates using homotopy analysis method, *Commun Nonlinear Sci Numer Simul* 2009; 14(5): 1868–1880.
- [28] Sohoulou AR, Famouri M, Kimiaieifar A, Domairry G. Application of homotopy analysis method for natural convection of Darcian fluid about a vertical full cone embedded in porous media prescribed surface heat flux, *Commun Nonlinear Sci Numer Simul* 2010; 15(7): 1691–1699
- [29] Ziabakhsh Z, Domairry G, Ghazizadeh HR. Analytical solution of the stagnation-point flow in a porous medium by using the homotopy analysis method, *Taiwan Institute Chem Eng* 2009; 40(1): 91–97.
- [30] Rashidi MM, Domairry G, Dinarvand S. Approximate solutions for the Burger and regularized long wave equations by means of the homotopy analysis method, *Commun Nonlinear Sci Numer Simul* 2009; 14(3): 708–717.
- [31] Rashidi MM, Mohimaniyanpour SA, Abbasbandy S. Analytic approximate solutions for heat transfer of a micropolar fluid through a porous medi-

- um with radiation, *Commun Nonlinear Sci Numer Simul* 2011; 16(4): 1874-1889.
- [32] Abdoul R. Ghotbi, H. Bararnia, G. Domairry, A. Barari: Investigation of a powerful analytical method into natural convection boundary layer flow, *Commun Nonlinear Sci Numer Simul* 2009; 14(5): 2222-2228
- [33] Bararnia H, Ghasemi E, Domairry G, Soleimani S. Behavior of micro-polar flow due to linear stretching of porous sheet with injection and suction, *Adv Eng Softw* 2010; 41(6): 893-897.
- [34] Jalaal M, Bararnia H, Domairry G. A series exact solution for one-dimensional non-linear particle equation of motion, *Powder Technol* 2011; 207 (1-3): 461-464.
- [35] Kimiaefar A, Saidi AR, Bagheri GH, Rahimpour M, Domairry DG. Analytical solution for Van der Pol-Duffing oscillators. *Chaos Solitons & Fractals* 2009; 42(5): 2660-2666.
- [36] Oztop HF, Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, *Int J Heat Fluid Flow* 2008; 29: 1326-1336.